• Arithmetic Operations

① Scalar Multiplication

The scalar product of a number k and a matrix A is the matrix denoted by kA obtained by multiplying each entry of A by the number k

The number k is called a scalar

$$4 \times \begin{bmatrix} 10 & -9 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 4\times10 & 4\times-9 \\ 4\times-1 & 4\times4 \end{bmatrix} = \begin{bmatrix} 40 & -36 \\ -4 & 16 \end{bmatrix}$$

② Addition

To add or subtract matrices, they must both be of the same order ◎ (size), m×n

To add matrices, add corresponding entries

$$[A+B] = [a_{ij} + b_{ij}]$$

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(3) **Subtraction**

To subtract matrices, they must be of the same order (size), $m \times n$.

To subtract matrices, subtract their corresponding entries.

$A - B = [a_{ij} - b_{ij}]$

Properties

Theorem Let A, B and C be $m \times n$ matrices. Let O denote the $m \times n$ zero matrix (all entries are zero). Then,

(1) $A + B = B + A$ (communicative law of addition)

(2) $A + (B + C) = (A + B) + C$ (associative law)

(3) $A - A = O = -A + A$

(4) If k is a number, then $k(A + B) = kA + kB$

$k(A - B) = kA - kB$.

Transpose of a Matrix

When the columns and the rows of a matrix are interchanged.

$$A = \begin{bmatrix} 5 & 6 & 5 \\ 9 & 8 & -4 \\ 3 & 5 & -3 \end{bmatrix} \qquad A^T \text{ or } A' = \begin{bmatrix} 5 & 9 & 3 \\ 6 & 8 & 5 \\ 5 & -4 & -3 \end{bmatrix}$$

Every matrix can be written as the sum of a symmetric and skew symmetric matrix

$$\text{matrix } A = \frac{A+A'}{2} + \frac{A-A'}{2}$$

$\frac{A+A'}{2}$ ↙ symmetric

$\frac{A-A'}{2}$ ↘ skew symmetric

Properties

The transpose of a transpose matrix, the matrix obtained is equal to the original matrix

$$(A')' = A$$

Transpose of an addition of 2 matrices A and B obtained is equal to the sum of the transpose of the individual matrices A and B.

$$(A+B)' = A' + B'$$

Transpose of the product of two matrices is equal to the product of the two matrices in reverse order

$$(AB)' = B'.A'.$$

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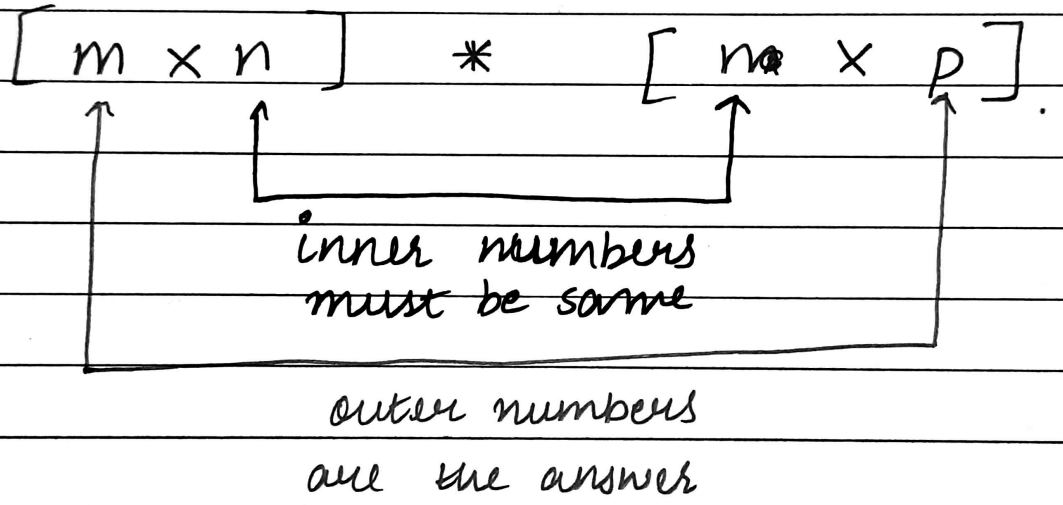
Multiplication of Matrices

The Matrices must be of the same size:

- no. of colums of first = number of rows of second
- when an $m \times n$ matrix is to be multiplied by $n \times p$.



The resulting matrix is of order = $\boxed{m \times p}$