SOP & POS Forms

STUDYBYNOTES

① SOP
(Sum of Product)

- Group of Product terms summed together.
- When two or more product terms are summed up Boolean addition.

SUM → OR (+)
Product → AND (·)

$Y = AB + AC + BC$

(SUM)

Product term

Examples:

$Y = AB + BC + AC$

$Y = \bar{P}\bar{Q} + PQR + PQ\bar{R}$

(Firstly Product then SUM)

(A, B, C, P, Q, R) → I/P terms

② POS
(Product of Sum)

- Group of sum terms Multiplied together.
- When two or more sum terms are product by Boolean multiplication

SUM → OR (+)
Product → AND (·)

$Y = (A+B) \cdot (B+C) \cdot (A+\bar{C})$

Product

Sum terms

Examples

$Y = (A+\bar{B}) \cdot (A+B) \cdot (A+\bar{C})$

$Y = (P+R) \cdot (P+\bar{Q}) \cdot (P+R)$

AND AND

OR OR OR

(i) Standard/Conorical SOP Form:- In this each product term consists of all the literals in the complemented or uncomplemented form.

e.g: (A, B, C) are three literals.

$$X = ABC + AB\bar{C} + \bar{A}BC$$

(ii) Standard/Canonical POS form:- In this each sum terms consists of all the literals in the complemented or uncomplemented form.

E.g:- $X = (A+B+\bar{C}) \cdot (\bar{A}+\bar{B}+C) \cdot (\bar{A}+\bar{B}+\bar{C})$

(iii) Minterm and Maxterm

- MINTERM:- Each individual term in the Standard SOP form is called minterm.

E.g:- $AB\bar{C} + A\bar{B}C + \bar{A}BC$ ↳ SOP
↳ minterms

It is represented by 'm'.

- MAXTERM:- Each individual term in the Standard POS form is called [maxterm.]

E.g:- $(A+B) \cdot (A+\bar{B})$ ↳ POS.
↳ Maxterms

represented by M.

⇒ Write minterms and maxterms for the following Truth Table.

A	B	minterms (SSOP) m_i	maxterms (SPOS) M_i
0	0	$\bar{A}\bar{B} \rightarrow m_0$	$A+B \rightarrow M_0$
0	1	$\bar{A}B \rightarrow m_1$	$A+\bar{B} \rightarrow M_1$
1	0	$A\bar{B} \rightarrow m_2$	$\bar{A}+B \rightarrow M_2$
1	1	$AB \rightarrow m_3$	$\bar{A}+\bar{B} \rightarrow M_3$

* Convert SOP to Standard SOP Form :-

(i) find each term find missing literal.
(ii) AND term with the term formed by ORing missing literal and its complement.

Example: $X = AB + A\bar{C} + BC$ [3 literals A, B, C]

$= AB \cdot (C+\bar{C}) + A\bar{C} \cdot (B+\bar{B}) + BC \cdot (A+\bar{A})$

$= ABC + AB\bar{C} + AB\bar{C} + A\bar{B}\bar{C} + ABC + \bar{A}BC$

[Y + X = X]

$= ABC + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}BC$

∴ This is standard SOP.

* Convert POS to Standard POS Term :-

(i) for each term, find missing literal.
(ii) OR each term with the formed by ANDing missing literal in that term with its complement.

Example :- $X = \underbrace{(A+B)}_{C}\ \underbrace{(A+C)}_{B}\ \underbrace{(B+\bar{C})}_{A}$ → 3 literals A, B, C

Property
$A+BC = (A+B)(A+C)$

$X = (A+B+C\bar{C}).(A+C+B\bar{B}).(B+\bar{C}+A\bar{A})$

$= (A+B+C).(A+B+\bar{C}).(A+C+B).(A+C+\bar{B}).$
$(B+\bar{C}+A).(B+\bar{C}+\bar{A}).$

$= (A+B+C).(A+B+\bar{C}).(A+\bar{B}+C).(\bar{A}+B+\bar{C}).$

This is standard POS Term.

(*) Represent the logical expressions with minterms and maxterms.

Minterms = m_i (Σ)

Maxterms = M_i (Π)

SOP :- 0 = Complement, 1 = Uncomplement
POS :- 0 = Uncomplement, 0 = Complement

Example ① $X = \underbrace{ABC}_{m_7} + \underbrace{\bar{A}BC}_{m_3} + \underbrace{A\bar{B}\bar{C}}_{m_4}$

A	B	C	m_i	M_i	
0	0	0	$\bar{A}\bar{B}\bar{C}$ m_0	$A+B+C$ M_0	
0	0	1	$\bar{A}\bar{B}C$ m_1	$A+B+\bar{C}$ M_1	
0	1	0	$\bar{A}B\bar{C}$ m_2	$A+\bar{B}+C$ M_2	
0	1	1	$\bar{A}BC$ m_3	$A+\bar{B}+\bar{C}$ M_3	1
1	0	0	$A\bar{B}\bar{C}$ m_4	$\bar{A}+B+C$ M_4	1
1	0	1	$A\bar{B}C$ m_5	$\bar{A}+B+\bar{C}$ M_5	
1	1	0	$AB\bar{C}$ m_6	$\bar{A}+\bar{B}+C$ M_6	
1	1	1	ABC m_7	$\overline{A+B+C}$ M_7	1

$$X = m_7 + m_3 + m_4$$

$$X = \Sigma m\,(3,4,7)$$

Example ② $(A+\bar{B}+C)\;(A+B+C)\;(\bar{A}+\bar{B}+C)$

$$X = (A+\bar{B}+C)(A+B+C)(\bar{A}+\bar{B}+C)$$

$$X = M_2 M_0 M_6$$

$$X = \Pi M\,(0,2,6)$$

$(A+\bar{B}+C)$	$(A+B+C)$	$(\bar{A}+\bar{B}+C)$
0 1 0	0 0 0	1 1 0
M_2	M_0	M_6

Write

* Standard SOP Expression for a given Truth Table:-

(i) Consider only input combinations whose output (X) is 1.

(ii) Write Product term for each such combination.

(iii) 'OR' all these Product terms

Example

A	B	C	X	Product
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	$\bar{A}BC$
1	0	0	0	
1	0	1	1	$A\bar{B}C$
1	1	0	1	$AB\bar{C}$
1	1	1	0	

Product terms

$$X = \underset{m_3}{\bar{A}BC} + \underset{m_5}{A\bar{B}C} + \underset{m_6}{AB\bar{C}}$$

$$X = m_3 + m_5 + m_6$$

$$X = \sum m(3,5,6)$$

* Write Standard POS Expression of a given Truth Table

(i) Consider only those combinations of Input which Produces low output (0)

(ii) Write maxterms only for such I/p combinations

(iii) 'AND' these maxterms.

Example:-

A	B	C	X	
0	0	0	0	→ $A+B+C$
0	0	1	1	
0	1	0	1	
0	1	1	0	→ $A+\bar{B}+\bar{C}$
1	0	0	1	
1	0	1	0	→ $\bar{A}+B+\bar{C}$
1	1	0	0	→ $\bar{A}+\bar{B}+C$
1	1	1	1	

Max terms

Standard POS $X = (A+B+C).(A+\bar{B}+\bar{C}).(\bar{A}+B+\bar{C}).(\bar{A}+\bar{B}+C)$

$M_0 \quad M_3 \quad M_5 \quad M_6$

$X = \prod M\,(0,3,5,6)$

$X = M_0.M_3.M_5.M_6$