

SOP & POS Forms



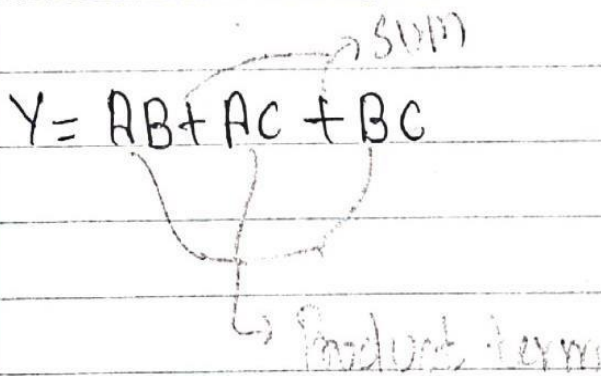
① SOP

(Sum of Product)

- Group of product terms ^{summed} together.
- When two or more product terms are summed up Boolean addition.

SUM \rightarrow OR (+)

Product \rightarrow AND (·)



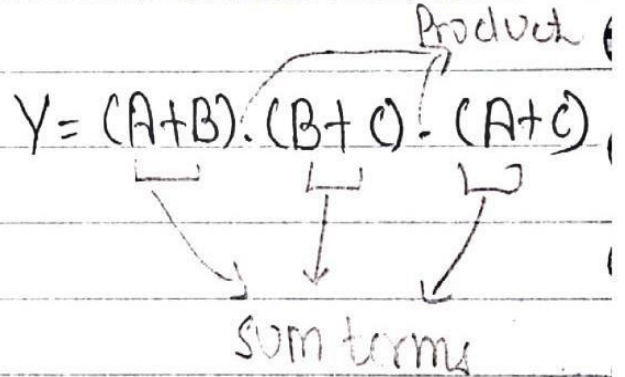
② POS

(Product of Sum)

- Group of sum terms multiplied together.
- When two or more sum terms are product by Boolean multiplication.

SUM \rightarrow OR (+)

Product \rightarrow AND (·)



Examples:

$$Y = AB + BC + AC$$

$$Y = \bar{P}\bar{Q} + PQR + P\bar{Q}\bar{R}$$

(A, B, C, P, Q, R) \rightarrow IP terms

Examples

$$Y = (A+\bar{B}) \cdot (A+B) \cdot (A+C)$$

$$Y = (P+R) \cdot (P+\bar{Q}) \cdot (P+R)$$

\downarrow \downarrow \downarrow
 OR OR OR

Firstly Product then Sum

(i) Standard/Canonical SOP Form:- In this each product term consists of all the literals in the complemented or uncomplemented form.
e.g. (A, B, C) are three literals.

$$X = ABC + AB\bar{C} + \bar{A}BC$$

(ii) Standard/Canonical POS form:- In this each sum term consists of all the literals in the complemented or uncomplemented form.

$$\underline{\text{E.g.}}:- X = (A+B+\bar{C}) \cdot (\bar{A}+B+C) \cdot (\bar{A}+\bar{B}+\bar{C})$$

(iii) Minterm and Maxterm



• MINTERM:- Each individual term in the Standard SOP form is called minterm.

$$\underline{\text{E.g.}}:- \underbrace{ABC} + \underbrace{AB\bar{C}} + \underbrace{\bar{A}BC} \text{ \& } \text{SOP} \\ \hookrightarrow \text{minterms}$$

It is represented by 'm'.

• MAXTERM:- Each individual term in the Standard POS form is called maxterm.

$$\underline{\text{E.g.}}:- \underbrace{(A+B)} \cdot \underbrace{(A+\bar{B})} \text{ \& } \text{POS.} \\ \hookrightarrow \text{maxterms}$$

represented by M.

→ Write minterms and maxterms for the following Truth Table.

		(SSOP)	(SPOS)
A	B	minterms m_i	maxterms M_i
0	0	$\bar{A}\bar{B} \rightarrow m_0$	$A+B \rightarrow M_0$
0	1	$\bar{A}B \rightarrow m_1$	$A+\bar{B} \rightarrow M_1$
1	0	$A\bar{B} \rightarrow m_2$	$\bar{A}+B \rightarrow M_2$
1	1	$AB \rightarrow m_3$	$\bar{A}+\bar{B} \rightarrow M_3$

* Convert SOP to Standard SOP Form:-

- (i) find each term find missing literal.
- (ii) AND term with the term formed by ORing missing literal and its complement.

Example! $X = AB + A\bar{C} + BC$ [3 literals]

$$= AB \cdot (C + \bar{C}) + A\bar{C} \cdot (B + \bar{B}) + BC \cdot (A + \bar{A})$$

$$= \underbrace{ABC} + \underbrace{AB\bar{C}} + \underbrace{A\bar{B}\bar{C}} + \underbrace{A\bar{B}C} + \underbrace{ABC} + \underbrace{\bar{A}BC}$$

[$X + X = X$]

$$= ABC + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}BC$$

This is standard SOP.

* Convert POS to Standard POS Term :-

- (i) for each term, find missing literal.
- (ii) OR each term with the formed by ANDing missing literal in that term with its complement.

Example:- $X = \underbrace{(A+B)}_C \underbrace{(A+C)}_B \underbrace{(B+\bar{C})}_A \rightarrow 3 \text{ literals } A, B, C$

Property
$A+BC = (A+B)(A+C)$

$$\begin{aligned}
 X &= (A+B+C\bar{C}) \cdot (A+C+B\bar{B}) \cdot (B+\bar{C}+A\bar{A}) \\
 &= \underbrace{(A+B+C)} \cdot \underbrace{(A+B+\bar{C})} \cdot \underbrace{(A+C+B)} \cdot \underbrace{(A+C+\bar{B})} \cdot \underbrace{(B+\bar{C}+A)} \cdot \underbrace{(B+\bar{C}+A)} \\
 &= (A+B+C) \cdot (A+B+\bar{C}) \cdot (A+B+C) \cdot (A+\bar{B}+\bar{C})
 \end{aligned}$$

This is standard POS Term.

(*) Represent the logical expressions with minterms and maxterms.

$$\text{Minterms} = m_i (\Sigma)$$

$$\text{Maxterms} = M_i (\Pi)$$

SOP \Rightarrow 0 = Complement, 1 = Uncomplement
 POS \Rightarrow 0 = Uncomplement, 1 = Complement

Example 1 $X = \underbrace{ABC}_{m_7} + \underbrace{\bar{A}BC}_{m_3} + \underbrace{A\bar{B}\bar{C}}_{m_4}$

A	B	C	m_i	M_i	
0	0	0	$\bar{A}\bar{B}\bar{C}$ m_0	$A+B+C$ M_0	
0	0	1	$\bar{A}\bar{B}C$ m_1	$A+B+\bar{C}$ M_1	
0	1	0	$\bar{A}B\bar{C}$ m_2	$A+\bar{B}+C$ M_2	
0	1	1	$\bar{A}BC$ m_3	$A+\bar{B}+\bar{C}$ M_3	1
1	0	0	$A\bar{B}\bar{C}$ m_4	$\bar{A}+B+C$ M_4	1
1	0	1	$A\bar{B}C$ m_5	$\bar{A}+B+\bar{C}$ M_5	
1	1	0	$AB\bar{C}$ m_6	$\bar{A}+\bar{B}+C$ M_6	
1	1	1	ABC m_7	$\bar{A}+\bar{B}+\bar{C}$ M_7	1

$$X = m_7 + m_3 + m_4$$

$$X = \sum m(3, 4, 7)$$

Example 2

$$(A + \bar{B} + C) (A + B + C) (\bar{A} + \bar{B} + C)$$

$$X = (A + \bar{B} + C) (A + B + C) (\bar{A} + \bar{B} + C)$$

$$X = M_2 M_0 M_6$$

$$X = \prod M(0, 2, 6)$$



$(A + \bar{B} + C)$	$(A + B + C)$	$(\bar{A} + \bar{B} + C)$
0 1 0	0 0 0	1 1 0
m_2	m_0	m_6

Write
 * a Standard SOP Expression for a ^{given} Truth Table -

(i) Consider only input combinations whose output (X) is 1.

(ii) Write Product term for each such combination.

(iii) 'OR' all these Product terms.

Example

A	B	C	X	Product
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	$\bar{A}BC$
1	0	0	0	
1	0	1	1	$A\bar{B}C$
1	1	0	1	$AB\bar{C}$
1	1	1	0	

} Product terms

$$X = \bar{A}BC + A\bar{B}C + AB\bar{C}$$

$m_3 \quad m_5 \quad m_6$

$$X = m_3 + m_5 + m_6$$

$$X = \sum m(3, 5, 6)$$

Standard

* Write POS Expression of a given Truth-Table

(i) Consider only those combinations of Input which produces low output (0)

(ii) Write maxterms only for such I/P combinations

(iii) 'AND' these maxterms.

Example:-

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$$\rightarrow A+B+\bar{C}$$

$$\rightarrow A+\bar{B}+\bar{C}$$

$$\rightarrow \bar{A}+B+\bar{C}$$

$$\rightarrow \bar{A}+\bar{B}+C$$

Max terms

Standard POS $X = (A+B+\bar{C}) \cdot (A+\bar{B}+\bar{C}) \cdot (\bar{A}+B+\bar{C}) \cdot (\bar{A}+\bar{B}+C)$

$\downarrow \quad \quad \downarrow \quad \quad \downarrow \quad \quad \downarrow$

$M_0 \quad \quad M_3 \quad \quad M_5 \quad \quad M_6$

$$X = \prod M (0, 3, 5, 6)$$

$$X = M_0 \cdot M_3 \cdot M_5 \cdot M_6$$

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