Non-redundant and Minimal Covers

F : given set
↓
G^+ : minimal set

A minimal cover of a set of functional dependencies (FD) 'F', is a set of FD's 'G', that satisfy the property that every dependency in F is in G^+. Then G is called minimal procedure to find minimal set.

Steps to find Minimal Set

(I) Split the FDs such that R.H.S. contain single attribute.
R.H.S. → Right hand side

E.g. $A \to BC$
$\Rightarrow A \to B$ & $A \to C$

(II) Find the redundant FDs and delete them from the set
repeat

E.g. $\{A \to B, B \to C, A \to C\}$ redundant
$\to \{A \to B, B \to C\}$

(III) find the redundant attributes on L.H.S. and delete them

E.g.:- $AB \to C$

(A can be deleted, if B^+ Contains A)
(B can be deleted, if A^+ contains B)

if there was any removal of variables in step3 repeat 2.

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Example

To find the Minimal Cover.

$\{A \to B, C \to B, D \to ABC, AC \to D\}$.

Step 1:- $\{A \to B, C \to B, D \to ABC, AC \to D\}$

(Decompose)

$\Rightarrow$

$A \to B, C \to B, D \to A, D \to B, D \to C, AC \to D$

Step 2: Remove Redundant FD's.

$A \to B, C \to B, D \to A, \boxed{D \to B}, D \to C, AC \to D$

redundant

firstly we get the closure's

$A^+ = A$

$C^+ = C$

$D^+ = DBC$

$D^+ = DABC$

$D^+ = DAB$

$AC^+ = ACB$

So $\to \{A \to B, C \to B, D \to A, D \to C, AC \to D\}$

Step 3: $A \to B, C \to B, D \to A, D \to C, AC \to D\}$

(We can't reduce) $\to$

$$\boxed{\begin{array}{l} AC \to D \\ C^+ = C \\ A^+ = AB \end{array}}$$

Ans:- $\{A \to B, C \to B, D \to AC, AC \to B\}$

Canonical Cover.

- A canonical cover is a simplified and reduced version of the given set of functional dependencies
- It is reduced version, it is also called as irreducible set.

Characterstics:-

- It is not unique, it may be more than one cover possible.
- Closure of Canonical cover = Intial FD's set Closure.
- Canonical Cover is free from all the $\boxed{\text{extraneous}}$ FDs.

Extraneous Attribute

An attribute of a functional dependency is said to be extraneous if we can remove it without changing the closure of the set of functional dependencies.

Example $R = (A, B, C)$ $FD = \{A \to BC, A \to B, AB \to C\}$

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here $A \to BC$ $\quad A \to B$ $\quad$ [SO $A \to B$ extra]
$A \to C$ $\quad$ because already there

$AB \to C$ $\quad$ A is extra $\left[\begin{matrix} A \to B \\ B \to C \end{matrix} \therefore A \to C \right]$.

because $B \to C$ already there

$A \to BC$: C is extra
by removing C, $A \to B$

Set $\{A \to B, B \to C\}$ all the Conditions satisfied.
↳ Canonical Cover's

Example to find Canonical form

$R(w, x, y, z)$

$F : \{x \to w,$
$wz \to xy,$
$y \to wxz\}$

Sol: Step1 :- (first we will check, if there is any extra attribute for each FD at R.H.S)

$x \to w$
$wz \to x$
$wz \to y$

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$Y \to W$

$Y \to x$

$Y \to Z$

Now Computre

$x^+ = \{x, W\}$

$x^+ = x$ (without using $x \to W$)

(so $x \to W$ is essential.

$WZ^+ = WZxy$

$WZ^+ = WZyx$ (without using $WZ \to x$)

($WZ \to x$ is not essential)

because FD, WZ^+ is same)

$WZ^+ = WZ$ (without using $WZ \to Y$)

it is essential.

$y^+ = \{y, W, x, Z\}$

$y^+ = y, x, Z, W$ (without $y \to W$) not essential

$y^+ = yZ$ (" $Y \to X$) essential

$y^+ = yXW$ (" $Y \to Z$) essential

All essential FDs

$X \to W$

$WZ \to Y$

$Y \to x$

$Y \to Z$

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Step 2:-

Now we will check if there is any extra attribute at left side of FD.

$WZ \to Y$

Compute $WZ^+ = WZYX$

$W^+ = W$

$Z^+ = Z$

$WZ \to Y$ is essential because W^+, Z^+ are different.

So:-

F_c :- $\{ X \to W,$

$WZ \to Y,$

$\left.\begin{array}{l} Y \to X \\ Y \to Z \end{array}\right\} \Rightarrow Y \to XZ \}$ is Canonical Set.