

Sorting

Sorting is the process of arranging the elements of an array so that they can be placed either in ascending or descending order.

Sorting is of two types :-

1. Internal Sorting
2. External Sorting

1. Internal Sorting :-

When the entire collection of data is very small so that sorting can take place in the main memory of the computer, it is called Internal Sorting.

2. External Sorting :-

- External Sorting is a type of sorting which can be implemented with the use of Secondary Storage.
- External Sorting is best when data is large enough that cannot be fit in the Primary Storage.

(2) Evaluation of Arithmetic Expressions:-

- An Arithmetic expression consists of operands and operators.
- In addition to operands and operators, the arithmetic expression may also include parenthesis like "left parenthesis" and "right parenthesis".

Example :- $A + (B - C)$

- Precedence Rules for Arithmetic Expression:-

Operators	Associativity	Precedence
\wedge exponentiation	Right to left	Highest followed by $*$ Multiplication and $/$ division.
$*$ $/$	Left to Right	Highest followed by $+$, $-$
$+$ $-$	Left to Right	Lowest

Evaluation of Arithmetic Expressions requires two steps:-

ci) first, convert the given expression into special notation.

cii) Evaluate the expression in this new notation.

Notations for Arithmetic Expressions:-

ci) Infix Notation.

cii) Prefix Notation.

ciii) Postfix Notation.

Infix Notation:-

The operator symbol is placed between the two operands.

$A+B$

$C*D$

E/F

$G-H$

Example: $A+B * C - D / F$

We can also use the concepts of round brackets along with the operands and operator to specify what operation is to be performed first along with the precedence level.

$(A+B) * (C-D)$

Prefix notation (Polish Notation):-

It refers to the notation in which the operator

is placed before its two operands.

$$+AB \quad *CD$$

We can easily convert the infix notation to the Polish Notation, step by step, using the concept of square brackets [].

$$(A+B) * (C-D) = [+AB] * [-CD] \\ = *+AB-CD$$

Reverse Polish Notation (RPN)

→ In this type of expression, operators comes after two operands.

$$AB+ \quad CD*$$

→ It is also called Postfix Notation.

→ We can easily convert the infix Notation to the Postfix Notation, step by step using the concept of square bracket [].

$$(A+B) * (C-D) \\ = [AB+] * [CD-] \\ = AB+CD*$$

* Evaluation of Postfix Expression:-

When an arithmetic expression is presented in the postfix form, you can use a stack to evaluate it to get value.

→ find the first operator from left to right and perform the operation.

→ It is free from any precedence.

Example:- $a b c * -$

Here first operator is $*$ therefore b and c is multiplied first.

$(b * c)$

Subtract a and multiplied operand to get in final result.

Example:- $4 6 8 * -$

Step 1:- $(6 * 8)$

Step 2:- $4 48 -$

Step 3:- $48 - 4$

Expression becomes 44

Algorithm:-

Start reading elements from left to right.

1. Read the next element.
2. If the element is operand then:-
push element in stack.
3. If element is operator - then
 - (a) pop two operands from stack
 - (b) Evaluate (perform operation).
 - (c) Push the result of expression into the stack top. \ominus
4. If no element then
pop the result
else
go to step 1.
5. Stop.

for example:-

$$P = 4, 5, 2, +, *, 16, 4, /, -$$

We can use above algorithm to evaluate it.

$P = 4, 5, 2, +, *, 16, 4, /, -$

Symbol	Scan	Stack
	4	4
	5	4, 5
	2	4, 5, 2
	+	4, 7
	*	28
	16	28, 16
	4	28, 16, 4
	/	28, 4
	-	24
Result :-		24

③ Conversion: from INFIX to POSTFIX Notation:-

To convert infix expression to postfix expression we will use the stack data structure.

Input and Output

Input:-

The infix expression $x^y / (5 * z) + 2$

Output:-

Postfix form is :- $xy^5z*/2+$

Algorithm:-

Step 1:- Push "(" on stack and ")" to end of infix expression.

Step 2:- Scan infix expression from left to right and repeat following steps until stack is empty.

Step 3:- If an operand is found add to P. (Postfix).

Step 4:- If an operator is found, then:-
(a) repeatedly pop all the operator from stack which are having higher or same precedence and put them into the P.

(b) Push the new operator to stack.

Step 5:- If left parenthesis is found push on stack.

Step 6:- If right parenthesis is found, then:-
(a) repeatedly pop the stack and add the popped operators to the expression P until a left parenthesis is found.
(b) Remove the left parenthesis and don't add it in P.

Step 7:- Stop

Example: Infix Notation $(P/(Q-R) * S + T)$

Symbol	Scan	Stack	Postfix Expression
((
P		(P
/		(/	P/
((/(P/(
Q		(/(P/(Q
-		(/(-	P/(Q-
R		(/(-	P/(Q-R
)		(/	P/(Q-R-
*		(/*	P/(Q-R-/*
S		(/*	P/(Q-R-/*S
+		(+	P/(Q-R-/*S+
T		(+	P/(Q-R-/*S+T
)			P/(Q-R-/*S+T+

Postfix Expression: - $PQR- /S * T+$

④ Parenthesis Matching:-

- A bracket is considered to be any one of the following characters: (,), {, }, [, or].
- Two brackets are considered to be a matched pair if the an opening bracket (i.e. [, {, or () occurs to the left of a closing bracket (i.e.), }, or).

- Matched pairs of brackets : $[\]$, $\{ \}$, and $()$.
- $\{ [()] \}$ is not balanced brackets. we can use a stack to store the unmatched brackets.

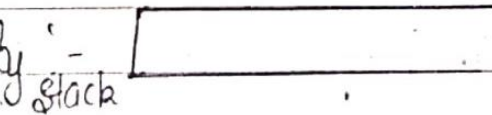
Algorithm:-

Scan the string from left to right,
and for each char :-

1. If a left bracket, push onto stack.
2. If a right bracket, pop bracket from stack.
(if not match or stack empty then fail).

Example:-

Initially -



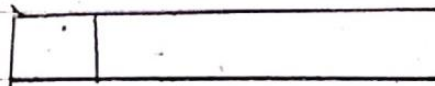
str

[{	()	}]
---	---	---	---	---	---

Opening bracket.
Push into Stack.

Step 1:-

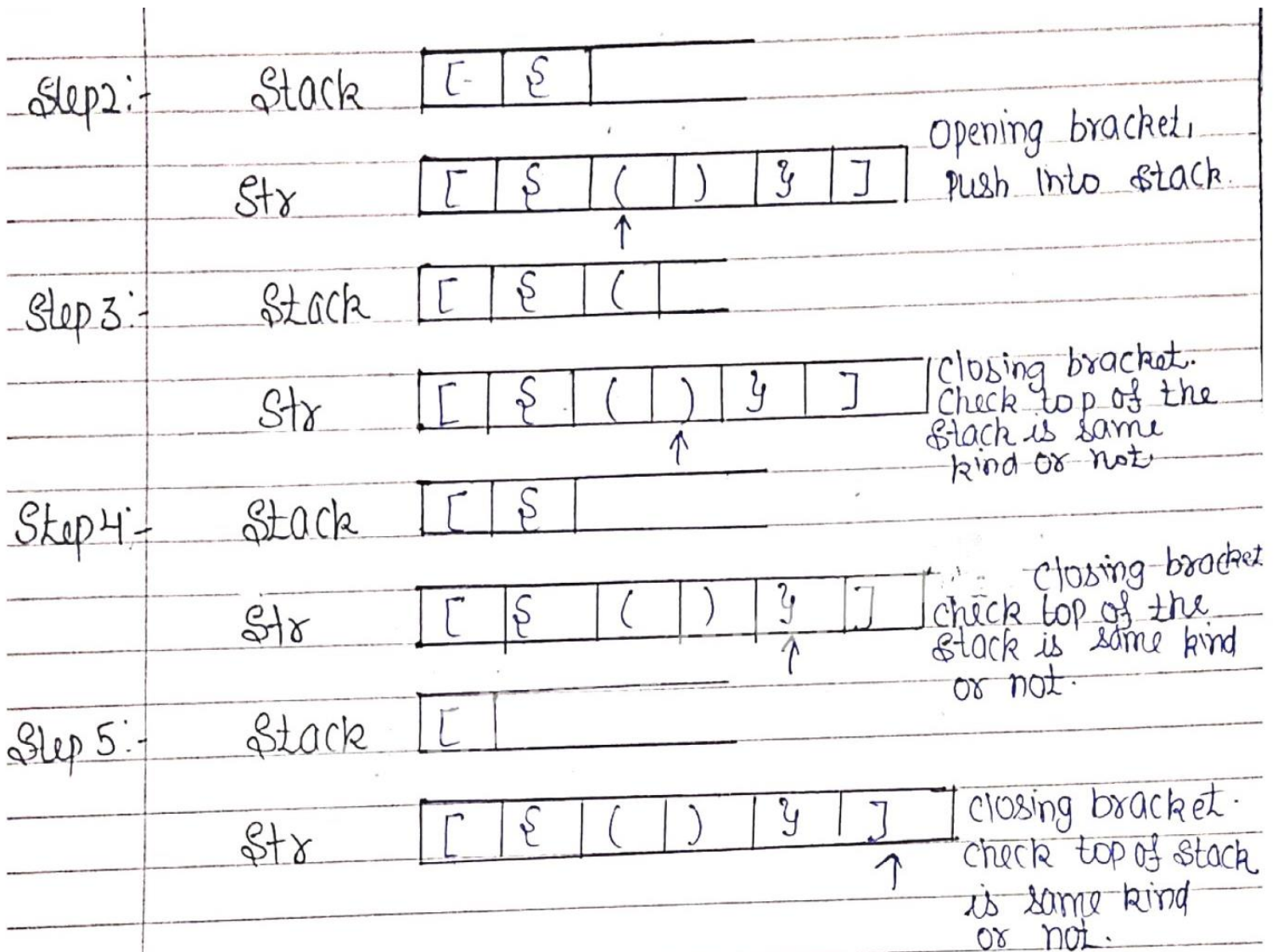
Stack



str

[{	()	}]
---	---	---	---	---	---

Opening bracket
push into Stack.



⑤ Recursion:-

- Recursion is a process of calling the function by itself and function is called recursive function.
- We can easily describe the implementation of recursion an application of stack using some of examples that use the concept of recursion.

Example of Recursion:-

1. factorial of positive integer.
2. Towers of Hanoi

1. factorial of a Positive integer:-

factorial is a mathematical term.

factorial of a number, say n , is equal to the product of all integers from 1 to n .

factorial of n is denoted by:-

$$n! = 1 \times 2 \times 3 \dots \times n.$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

$$\text{fact}(n) = \begin{cases} 1 & \text{if } n=0 \text{ or } n=1 \\ n * (n-1)! & \text{if } n > 1 \end{cases}$$

Algorithm:- factorial (N).

Step 1 :- if $N=0$ then
Return (1)

else

Return ($N * \text{factorial}(N-1)$)

[end if]

Step 2 :- End.

2. Towers of Hanoi :-

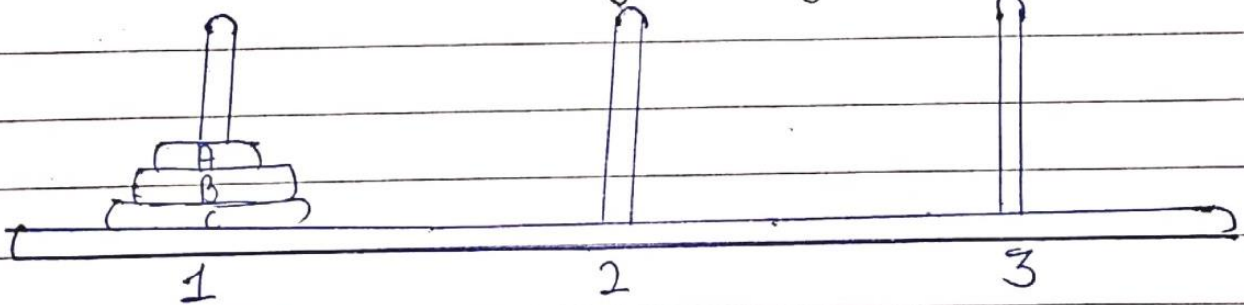
No discussion of recursion would be complete without the towers of Hanoi problem.

We are given a tower of N disks, initially stacked in increasing size on one of three poles.

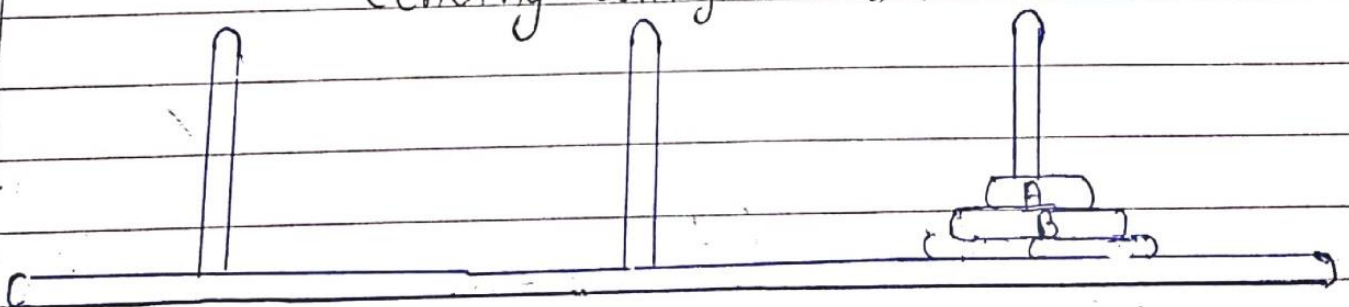
The objective is to transfer the entire tower to one of the other pole.

Rules:- (a) Move only one disc at a time.
(b) Only the top disc can be moved.
(c) A larger disc can not be placed on a smaller disc.

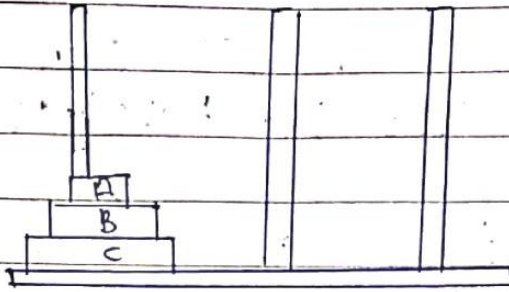
(Starting Configuration)



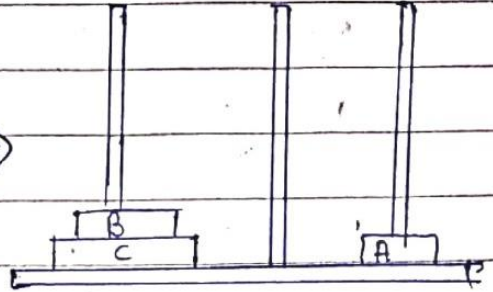
(Ending Configuration)



Solution:

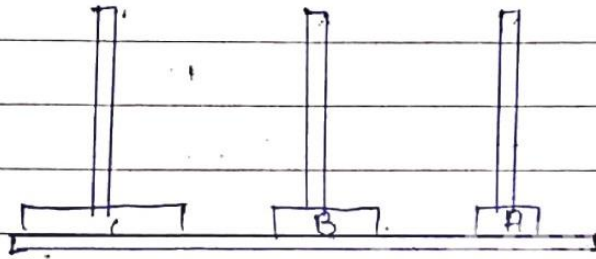


⇒

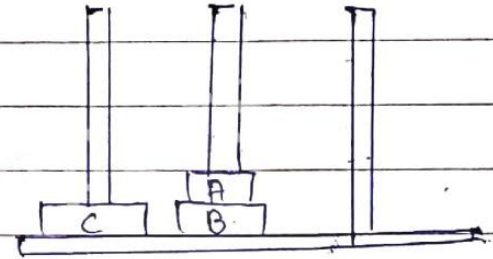


i) 1 → 3

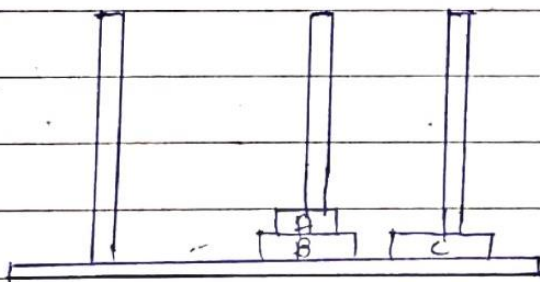
⇒



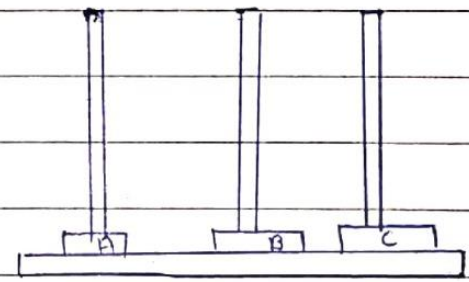
(ii) 1 → 2



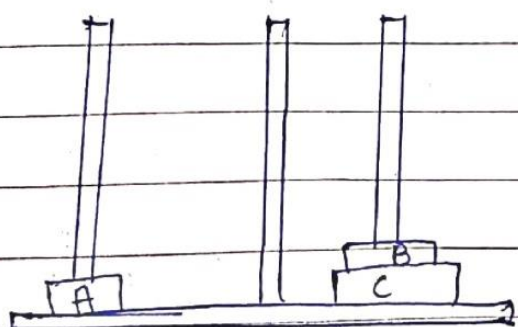
(iii) 3 → 2



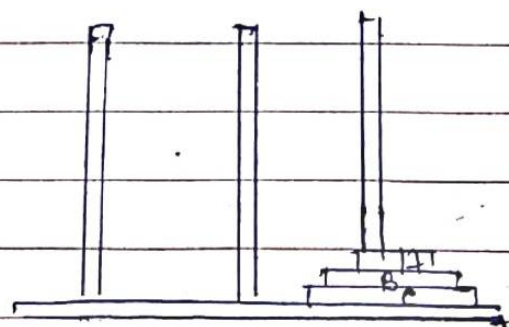
(iv) 1 → 3



(v) 2 → 1



(vi) 2 → 3



(vii) 1 → 3

