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27 march 19

* Isometric projections :-

$$\{ f_x = f_y = f_z \}$$

$$f_x^2 = f_y^2$$

$$\sin^2\phi = \frac{\sin^2\theta}{1-\sin^2\theta} \quad \text{—— (1)}$$

$$f_y^2 = f_z^2$$

$$\sin^2\phi = \frac{1-2\sin^2\theta}{1-\sin^2\theta} \quad \text{—— (2)}$$

equating (1) & (2)

$$\sin^2\theta = 1/3$$

$$\sin\theta = \pm\sqrt{1/3}$$

$$\therefore \sin^2\phi = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2}$$

$$\phi = 45^\circ$$

Substituting ϕ and θ in $R_y(\phi) R_x(\theta)$ gives us req. rotation matrices.

→ There are 4 possible values of isometric projections

↓

$\pm\theta, \pm\phi$ } combinations

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To find foreshortening factors

$$f_x = \sqrt{\cos^2\theta} = \sqrt{2/3}$$
$$= \underline{0.8165}$$

i.e 0.81% is projected length out of the total length.

An isometric projection is a special case of a dimetric projection.

→ The angle that the projected x axis makes with the horizontal is imp. in manual construction of isometric projections.

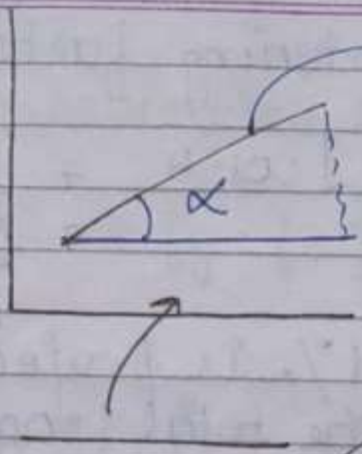
Transforming unit vector along x axis using isometric projection matrix yields.

$$[V_x^*] = [1\ 0\ 0\ 1] \underbrace{\begin{bmatrix} \cos\phi & \sin\phi\sin\theta & 0 & 0 \\ 0 & \cos\theta & 0 & 0 \\ \sin\phi & -\cos\phi\sin\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_y(\phi)\,R_x(\theta)\,P_z}$$

$$= [\cos\phi \quad \underline{\sin\phi\sin\theta} \quad 0 \quad 1]$$

Angle b/w projected x axis & horizontal.

$$\tan\alpha = \frac{y_x^*}{x_x^*} = \frac{\sin\phi\sin\theta}{\cos\phi}$$
$$= \underline{\pm \sin\theta}$$



line upon projection

i.e. $\alpha = \tan^{-1}$ $(\pm 35.26°)$

$= \pm 30°$

{angle of isometric projection}

EX3-15 (Book) $\phi = -45°$
$\theta = +35.26°$

Q (Derivation of Isometric, Dimetric & Trimetric projection matrix)

- Oblique Projections –
  parallel projection (C.O.P = ∞)
  projectors are parallel to each other but they are making an oblique angle (<90°) with POP.

  On basis of angle made

  cavalier   cabinet.

→ The face which is ∥ to POP, only that face will have its true shape & size.

Cavalier Projection

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- A cavalier projection is obtained when angle b/w oblique projectors & plane of projection is 45°

- Foreshortening factors for all 3 principal directions are equal. Thus resulting fig. appears too thick.

Cabinet Projection

- Cabinet projection is one for which foreshortening factors for edges ⊥ to POP is one-half.

(provided plane is $x-y$)

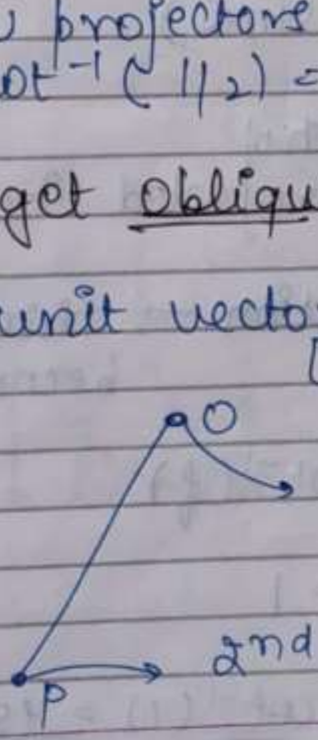
- lines ∥ to x & y foreshortens by an amt but lines ∥ to z axis gets foreshortened by $1/2$.

- angle b/w projectors and plane of projection is $\cot^{-1}(1/2) = 63.43°$.

* How to get Oblique projection matrix-

consider unit vector in z dir^n

$[0 \; 0 \; 1]$



1 projector passes from here

2nd projector passes from here.

→ If all projectors are made to meet at a pt., then they intersect by making an angle β which is same as angle b/w projected line & projector/unit vector.

(2) Translate pt. P by −a and −b.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -a & -b & 1 \end{bmatrix}$$

in 3D →

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -a & -b & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This becomes shearing matrix in 3-D

$$\left\{ \begin{array}{l} a = f\cos\alpha \\ b = f\sin\alpha \end{array} \right.$$

α → angle with horizontal axis.

f → length of projected line.

For cabinet projection → this f will become 1/2.

$$\beta = \cot^{-1}(f)$$

If f = 1

$$\beta = \cot^{-1}(1) = \underline{45^\circ}$$

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Oblique projection matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -f\cos\alpha & -f\sin\alpha & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{Bmatrix} \text{If } f = 1 \text{ (cavalier)} \\ \text{If } f = 1/2 \text{ (cabinet)} \end{Bmatrix}$$

- Perspective Transformations :-

C.O.P = finite distance
projectors are not parallel to each other i.e They meet at a pt.

If in its col → values are other than 0
then perspective, otherwise affine.
(lengths & angles are not preserved)

$$\begin{bmatrix} a & b & c & 0 \\ d & e & f & 0 \\ g & h & i & 0 \\ l & m & n & 1 \end{bmatrix}$$ → affine Transformation

$$\begin{bmatrix} & & & p \\ & & & 0 \\ & & & 0 \\ & & & 1 \end{bmatrix}$$

perspective Transformation.

$$\begin{bmatrix} & & 0 & p \\ & & 0 & 0 \\ & & 0 & 0 \\ & & 0 & 0 \end{bmatrix}$$

Projection on $z = 0$

perspective projection.

or perspective Transf. followed by orthographic Transformation.

$$\begin{bmatrix} & & & p \\ & & & q \\ & & & r \\ & & & 1 \end{bmatrix}$$

If any of $p, q, r \neq 0$
single pt. perspective Transformation.

Based on p, q, r values as 0, they tell where is centre of projection.

If $q \neq 0$ C.O.P lies along y axis.

If $p \neq 0$ C.O.P lies along x axis
If $r \neq 0$ " " " z axis.

finite dist. along one of the axis

$\underbrace{\text{finite dist. along one of the axis}}_{\text{Single pt. perspective Transformation.}}$

$$[x\ y\ z\ 1]\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & r\\ 0 & 0 & 0 & 1\end{bmatrix} = [x\ y\ z\ rz+1]$$

To homogenize coordinates, last value must be 1

$$[x^*\ y^*\ z^*\ 1] = \left[\frac{x}{rz+1}\ \ \frac{y}{rz+1}\ \ \frac{z}{rz+1}\ \ 1\right]$$

A perspective projection on $z=0$ is

$$[T] = [P_T][P_z]$$

orthographic projection

single pt perspective Transformation matrix

$$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & r\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[x \; y \; z \; 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= [x \quad y \quad 0 \quad rz+1]$$

$$[x^p \quad y^p \quad z^p] = \left[\frac{x}{rz+1} \quad \frac{y}{rz+1} \quad 0 \quad 1 \right]$$

If C.O.P — x_c

$$p = \frac{-1}{x_c}$$

If C.O.P — z_c

$$r = -\frac{1}{z_c}$$

If C.O.P — y_c

$$q = \frac{-1}{y_c}$$

* Vanishing point :-

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Perspective Projections

(gives less info. as seen from 1 pt.) 1 point — 2 point — 3 point. (most info. can be obtained)

4x4 Transformation 3-D

$$\begin{bmatrix} & & & 0 \\ & & & 0 \\ & & & 0 \\ & & & 1 \end{bmatrix}$$

Affine Transformation (|| lines remain parallel)

Other than these values
↓
Perspective Transformation

(last col. other than 0001)

→ $$\begin{bmatrix} & & & p \\ & & & q \\ & & & r \\ & & & 1 \end{bmatrix}$$

If $p \neq 0$ or $q \neq 0$ or $r \neq 0$
(1 point)
one value is non zero.

If 2 of these values are non-zero
(2 point)
$p \neq 0, r \neq 0, q = 0$
$p \neq 0, q \neq 0, r = 0$.

If all 3 are non zero (p, q, r) → (3 point).

Possible Directions

$$\underline{p = -\frac{1}{x_c}} \qquad \begin{bmatrix} p \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

C.O.P lies along +ve x axis. (x_c)

$$\underline{q = -\frac{1}{y_c}} \qquad \begin{bmatrix} 0 \\ q \\ 0 \\ 1 \end{bmatrix} \rightarrow$$ C.O.P lies along +ve y axis. (y_c)

$$\underline{r = -\frac{1}{z_c}} \qquad \begin{bmatrix} 0 \\ 0 \\ r \\ 1 \end{bmatrix} \rightarrow$$ C.O.P lies along z axis. (z_c)

Perspective Transformation followed by orthographic Transformation/projection along an axis.

Perspective ~~Transformation~~ Projection

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\underline{P_{T\,at\,z}}} \quad \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{P_z}$$

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→ After perspective Transformation, lines no longer remain parallel. Thus earlier pt. lying at ∞ but now the intersection pts. become visible

↓

∴ Vanishing pt.

(can be found using C.O.P)

If C.O.P → x_c

∴ Vanishing pt. → $-x_c$

If C.O.P → z_c

Vanishing pt. meets z axis at a particular pt. i.e $-z_c$

If C.O.P → y_c

Vanishing pt. → $-y_c$

* Principle Vanishing pt.

* 3 types of single pt. Perspective Transformation

↓

In Transformation → 3D pt. remains 3D

In Projection → 3D pt. becomes 2D (one col. becomes 0)

(1)
$$\begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Type I Transformation

C.O.P — x axis.

② along y axis (y_c)

③ along z axis (z_c)

eg:- If C.O.P → $(0, 0, -z_c, 1)$
Vanishing pt.
↓ $(0, 0, -z_c, 1)$

If given p, q, r we can find values of x_c, y_c & z_c

$$p = \frac{-1}{x_c}, \quad q = \frac{-1}{y_c}, \quad r = \frac{-1}{z_c}$$

· To get a better view, we have 2 pt. perspective Transformations
↓
2 non zero values
2 vanishing pts

$$\begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1 in opp. x dir^n & other in opp. y dir^n

$COP_x = (0, 0, -1/p, 1)$ ✗

$COP_y = (0, 0, -1/q, 1)$ ✗

→ $COP_x = (\overbrace{-1/p}^{x_c}, 0, 0, 1)$

→ $COP_y = (0, \underbrace{-1/q}_{y_c}, 0, 1)$

$$\underset{\text{object}}{[X]}\begin{bmatrix} 1 & 0 & 0 & 1/10 \\ 0 & 1 & 0 & 1/10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Orthographic Transformation along } z}$$

for a cube
↓
{8 vertices} Obj matrix → 8×4.

Orthographic Transformation along z.

Q Apply 3 pt. Transformation at $x_c = -10$, $y_c = -10$ and $z_c = 10$.
Projection is along $z = 0$ plane.

$p = 1/10$, $q = 1/10$, $r = -1/10$.

$$[X]\cdot\underbrace{\begin{bmatrix} 1 & 0 & 0 & 1/10 \\ 0 & 1 & 0 & 1/10 \\ 0 & 0 & 1 & -1/10 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{P_r}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \underbrace{0}_{\text{along } z} & 1 \end{bmatrix}$$

Q Translate an object and apply. 1 pt. Transformation projection at z axis.

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ l & m & 0 & 1 \end{bmatrix}}_{\text{Translation in 3D}}\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{1 pt. Transformation matrix}}$$

along x ← l

along y ← m