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13 feb, 19

Transformations

* 2-D Transformation :-
  ↓
  vertex is represented by 2 coordinates

→ Transformations are any geometric change applied to an obj. from current to next state.

→ Transformations help to modify obj. w/o redrawing once obj. is scan converted.

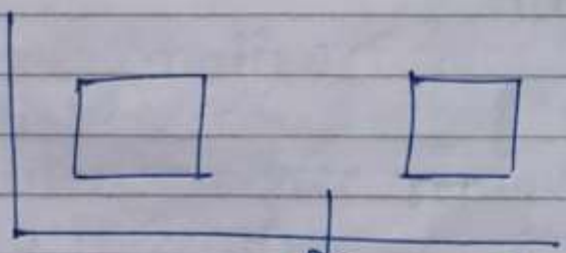
• 2 ways of Transformation.

① Object Transformation

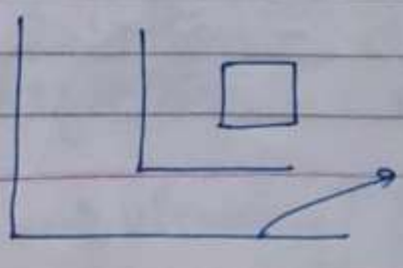
in syll.

Alter coordinates description
- Translation, rotation, scaling
- coordinate system unchanged.

② Coordinate Transformation
- Produce diff. coordinate system.



(i) Translation (coord. changed & sq. is drawn.

(ii) 
coordinate system changed but sq remains same pos.

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* Matrix Math → used for Transformations.

- more convinient organisation of data
  1 coord. → every row/col.

- more efficient processing
  no. of vertices = no. of rows.

- enables combination of various concatenations, rather than applying indivi- for series of transformation. dual transformations.

- There is a diff. b/w possible representations.

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e \\ f \end{bmatrix}$ → vertex in form of col vector
then we post multiply object with Transformation matrix

$[e \ f] \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

vertex is in form of row vector ∴ we pre multiply object with Transformation matrix.

each coordinate has 2 vertices.

For square
↓ 4 vertices.

$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

$[a \ b \ c \ d]$

→ order will be 2X4 (obj. matrix)

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can be derived from Basic transformations { Reflection → rotation by 180°. Shearing }.

Transformation matrix. (2X2)

$$[\;]_{2\times2} \cdot [\;]_{2\times4} = \underline{\underline{2\times4}}$$

$$[\;]_{4\times2} \cdot [\;]_{2\times2} = \underline{\underline{4\times2}}$$

NOTE: We will use column vector representation of a point.
i.e. pre multiplication of Transformation matrix / post multiplication of obj. matrix.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}}_{\text{resultant Transformed coordinate}}$$

① Translation.

- moving a pt. to a new position along same straight line.
- The path is represented by vector called translation/shift vector
- Translation factors: along x, along y

$$p'x = px + t_x \qquad p'y = py + t_y$$

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Basic Transformations

Rotation Scaling Translation.

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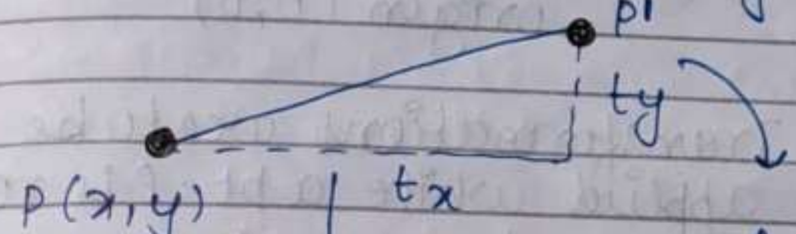
For diagonal Translation, changes will be in both x or y.



factor

add Translation of x along Px and y along Py.

$$P'x = px + tx$$
$$P'y = py + ty$$

In matrix form.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \underbrace{\begin{bmatrix} tx \\ ty \end{bmatrix}}$$

Translation factors are always added.

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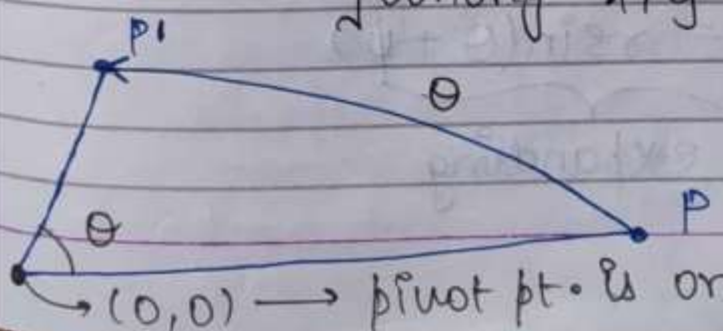
② **Rotation**

- repositions all pts. in an object along a circular path in plane centred at the pivot pt.

- rotation of a pt. means rotating its $\underbrace{\text{position vector.}}$

joining x, y to origin.



→ pivot pt. is origin.

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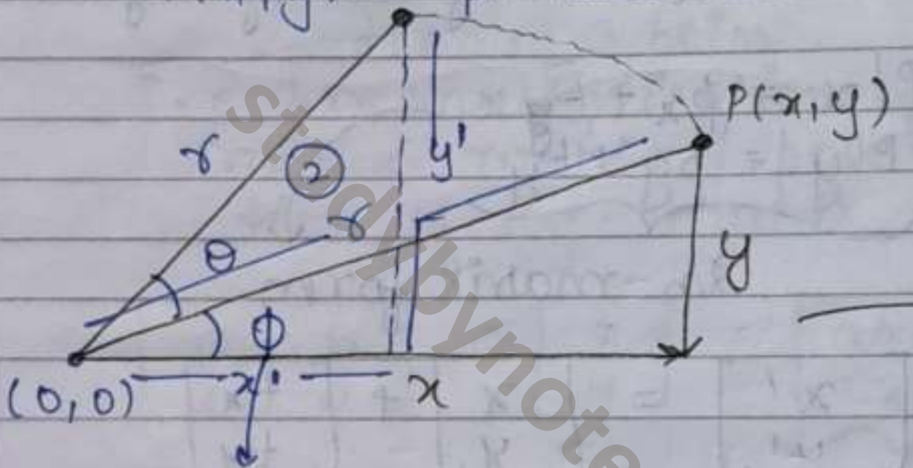
Rotation is always along a particular pt. If not mentioned, that pt. is origin (0,0)

* What Transformations are to be applied while a pt. P is rotated by angle θ.

→ $P(x,y)$ → pt. to be rotated in anticlockwise dir^n (+ve)



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angle that position vector p makes with axis.

$$\left\{ \begin{aligned} x &= r\cos\phi \\ y &= r\sin\phi \end{aligned} \right\}$$

We rotate P to a new pt $P'(x', y')$ by θ.

Acc. to Δ 2

$$\underbrace{\begin{aligned} x' &= r\cos(\theta+\phi) \\ y' &= r\sin(\theta+\phi) \end{aligned}}_{\text{expanding}}$$

$x' = r\cos\theta\cos\phi - r\sin\theta\sin\phi$

$x' = \underline{x\cos\theta} - y\sin\theta$ —— ①

$\left\{ \begin{array}{l} r\sin\phi = y \\ r\cos\phi = x \end{array} \right\}$

∴ θ is known & ϕ is unknown.

$\sin(\theta + \phi) = y'/r$

$y' = r\cos\phi\sin\theta + r\sin\phi\cos\theta$

$y' = \underline{x\sin\theta} + y\cos\theta$ —— ②

Here x' and y' are represented in terms of previous (x, y) coordinates.

Transformed coordinate $\left\{ \begin{array}{l} p'x = px\cos\theta - py\sin\theta \\ p'y = px\sin\theta + py\cos\theta \end{array} \right.$

↓

$P' = \underline{R \cdot P}$ (matrix form)

NOTE: θ can be clockwise (−ve) or counterclockwise (+ve).

$(\theta = -\theta)$ $(\theta = \theta)$

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} px \\ py \end{bmatrix}$$

If px and py is taken as row vector.

$$\underline{[px \;\; py]} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$ Transformation matrix changes.

↓

Transpose of matrix!

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NOTE:- When obj. matrix pos. changes, then Change pos. of Transformation matrix

Q (in exam) numericals (10-15 marks) + theory (10 marks)

Q Find transformed pt. P' caused by rotating P = (5, 1) abt origin through 90°.

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 5\cos 90 - 1\sin 90 \\ 5\sin 90 + 1\cos 90 \end{bmatrix}$$

new Transformed pt. $\left\{ = \begin{bmatrix} -1 \\ 5 \end{bmatrix} \right.$

3) **Scaling** - - Scaling changes the size of an object & involves 2 scale factors S_x and S_y for x & y coordinates.

- Scales are about origin.
- $\underbrace{\text{Scaling along x for a square}}$ changes into rectangle (same along y)

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If scaling factor along x & y is same

Uniform scaling.
(Shape doesn't change)

If scaling factors > 1
size ↑es
else ↓es.

size may ↑se/ ↓se depending upon S_x & S_y

- scaling is done around a pivot pt.

(normally origin)

$$\begin{cases} p'_x = S_x \cdot p_x \\ p'_y = S_y \cdot p_y \end{cases}$$

in matrix ⟶ $p' = S \cdot P$

scale matrix $S = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}_{2\times2}$

diagonal values are 0

as it is 2-D Transformation

$$\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{aligned} x' &= S_x \cdot x \\ y' &= S_y \cdot y \end{aligned}$$

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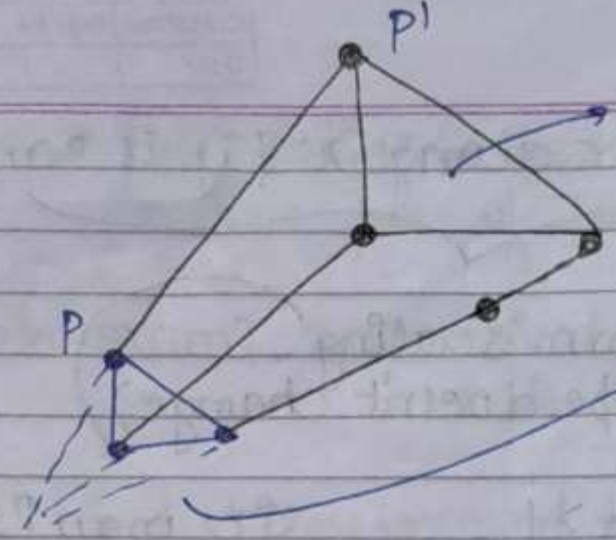
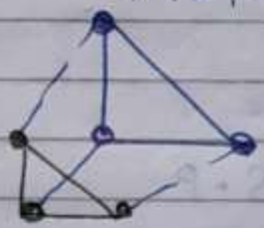


Scaling factors > 1

If scaling factors < 1

Q If scale factors are in b/w 0 and 1

pts. will be moved closer to origin i.e object becomes smaller



$S_x = 0.5$

$S_y = 0.5$

$$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

If scale factors > 1

pts. moved far from origin i.e object becomes larger

$S_x = S_y$ (uniform scaling) only change in size.

$S_x \neq S_y \rightarrow$ (differential scaling) change in size & shape.

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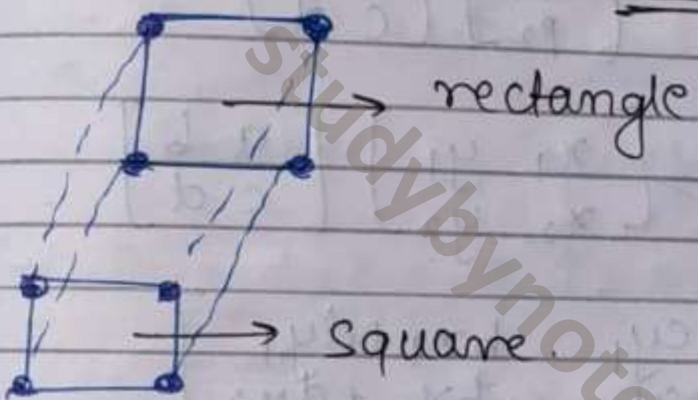
sq → rectangle

If $\underbrace{\text{scaling factors} = 1}$

size remains same.

If $\underbrace{\text{scaling factors are -ve}}$

???



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Transformation of Straight Line (Proof 1)

→ Any generalised 2-D Transformation transforms a S.L into a S.L

$$p_1 = [x_1, y_1], \quad p_2 = [x_2, y_2]$$

Transforming line with matrix

$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$p_1^* = p_1 T = [x_1 \;\; y_1] \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

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$= [ax_1 + cy_1 \quad bx_1 + dy_1]$

$P_2^* = P_2 T = [x_2 \; y_2] \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= [ax_2 + cy_2 \quad bx_2 + dy_2]$

Transformation of whole object

$\therefore$ This can be written as

$$\begin{bmatrix} P_1^* \\ P_2^* \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

row wise

$$= \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} ax_1 + cy_1 & bx_1 + dy_1 \\ ax_2 + cy_2 & bx_2 + dy_2 \end{bmatrix}$$

$$= \begin{bmatrix} x_1^* & y_1^* \\ x_2^* & y_2^* \end{bmatrix}$$

Transformed pts. are same as P_1^* & P_2^*

$\therefore$ SL always Transforms to a S.L

Proof of Transformation of SL

It can only be proved that every pt. on original line corresponds to a pt. in the new line

Take any pt. (mid pt.) on original line.

$\dfrac{x_1 + x_2}{2}, \quad \dfrac{y_1 + y_2}{2}$

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Transform mid pt. using TM

Find mid pt. of Transformed line & if they become equal. then it is prooved.

* mid pt. Transformation.

$$\underbrace{\frac{ax_1 + cy_1 + ax_2 + cy_2}{2}, \frac{bx_1 + dy_1 + bx_2 + dy_2}{2}}_{\text{mid pt. of Transformed line.}}$$

$$[x_1^* \quad y^*] = \left[\frac{x_1^* + x_2^*}{2} \quad \frac{y_1^* + y_2^*}{2}\right]$$

$$= \left[\frac{(ax_1 + cy_1) + (ax_2 + cy_2)}{2}, \frac{(bx_1 + dy_1) + (bx_2 + dy_2)}{2}\right]$$

$$= \left[a\frac{(x_1 + x_2)}{2} + c\frac{(y_1 + y_2)}{2} \quad b\frac{(x_1 + x_2)}{2} + d\frac{(y_1 + y_2)}{2}\right]$$

— ①

Transformation of original line

$$[x_1 \quad y_1]\, T = \left[\frac{x_1 + x_2}{2} \quad \frac{y_1 + y_2}{2}\right]\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \left[a\frac{(x_1 + x_2)}{2} + c\frac{(y_1 + y_2)}{2} \quad b\frac{(x_1 + x_2)}{2} + d\frac{(y_1 + y_2)}{2}\right]$$

same as ①

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