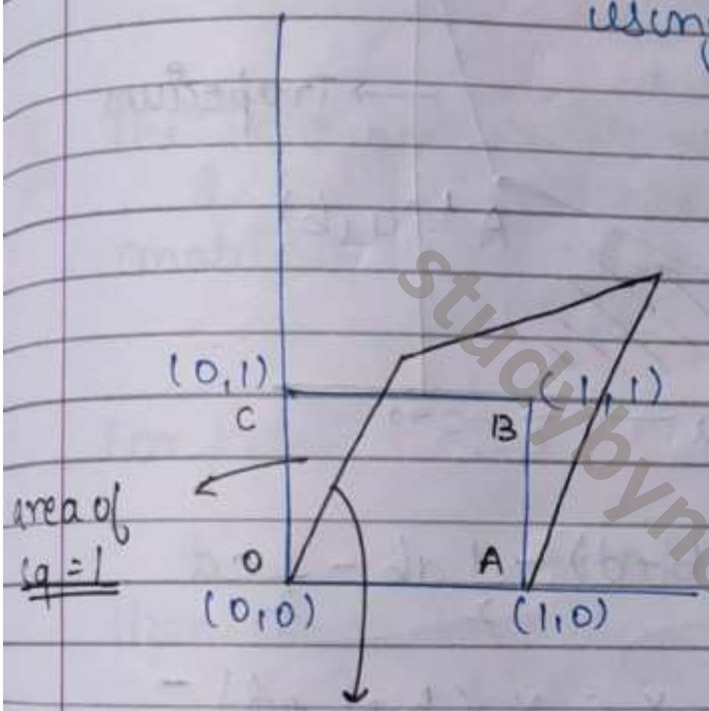
what happens to area when fig. is transformed using 2-D Transformation matrix.

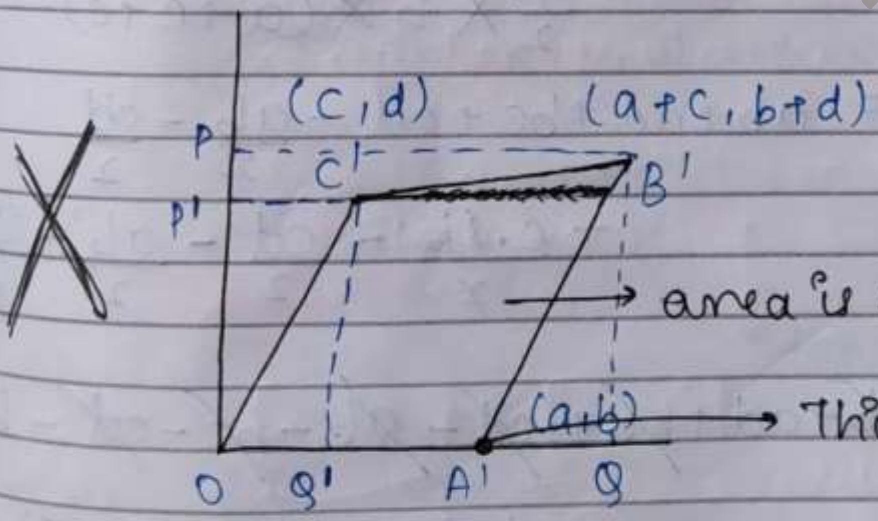
Consider a unit square. Transforming it using generalised 2-D matrix.



$$\begin{array}{c} O \\ A \\ B \\ C \end{array}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}_{(4\times 2)} \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{(2\times 2)}$$

$$\begin{bmatrix} 0 & 0 \\ a & b \\ a+c & b+d \\ c & d \end{bmatrix}_{(4\times 2)} \begin{array}{l} \rightarrow O \\ \rightarrow A' \\ \rightarrow B' \\ \rightarrow C' \end{array}$$

after Transformation



area is ??

This is not (a,b).

Area of $OA'B'C'$ = area of $OPB'Q$ − Area$(OP'C')$ − area$(PB'C'P')$ − area$(A'B'Q)$

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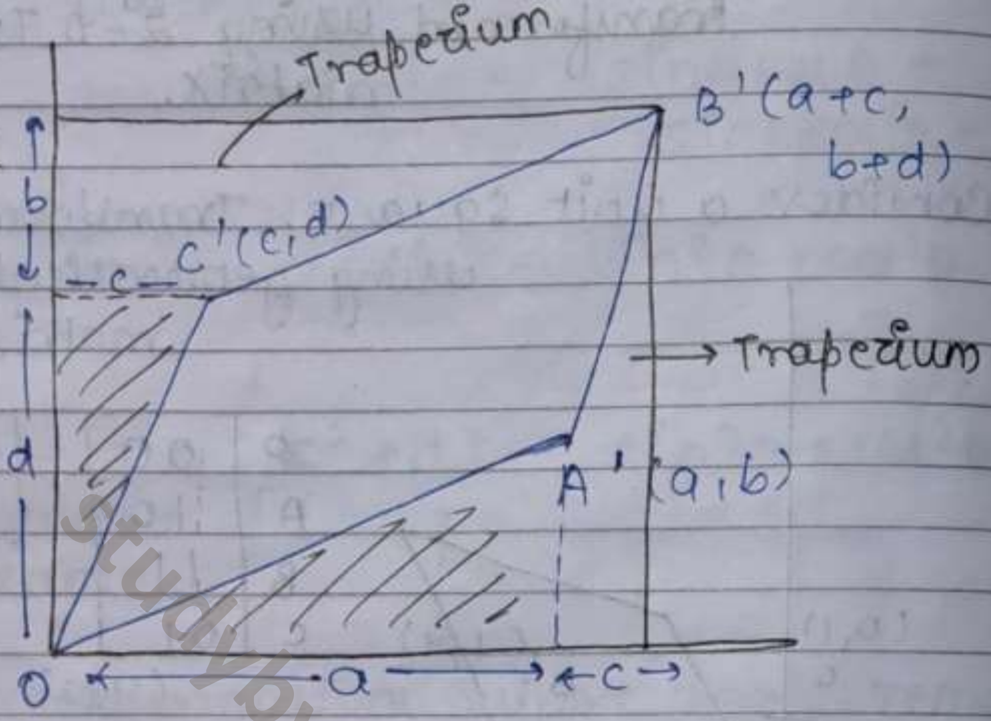
$ar(O'PB'Q) = \underline{\underline{(a+c)(b+d)}}$

Values of a, b, c, d are diff. Thus figure can't be predicted



$$\text{Area} = (a+c)(b+d) - \frac{1}{2}ab - \frac{1}{2}cd - \frac{1}{2} \times c \times (b+b+d) - \frac{1}{2} \times b \times (a+c+c)$$

$$= ab + ad + bc + cd - \frac{ab}{2} - \frac{cd}{2} - \frac{c.\cancel{2}b}{\cancel{2}} - \frac{cd}{2} - \frac{ab}{2} - \frac{\cancel{2}bc}{\cancel{2}}$$

$$= \cancel{ab} + ad + \cancel{bc} + \cancel{cd} - \cancel{ab} - \cancel{bc} - \cancel{cd} - bc$$

$$= \underbrace{\underline{\underline{ad - b \cdot c}}} = \underline{\underline{\det(T)}}$$

acc. to Transformation matrix, this is ~~Δ~~ det of matrix

→ Area of Transformed fig → $1 \times \det(T)$

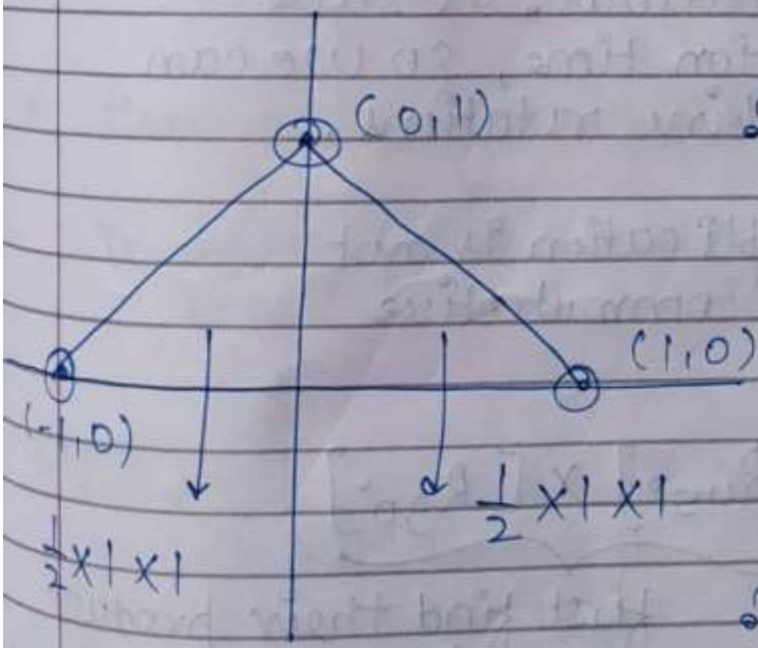
→ Area of any figure is ↓

$\boxed{\text{area of original fig} \times \det(T)}$

Q The Triangle with vertices $(1,0)$, $(0,1)$ & $(-1,0)$ is transformed using a matrix $\begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$. Find area of Transformed fig.

For proof
Find area of original matrix.

Then transform coordinates, and again find its area. Both will be Same.



∴ area of Δ
$= \frac{1}{2} + \frac{1}{2}$

$= 1$

$|\det| = 6 + 2$
$= 8$

∴ area of Transformed fig $= 8$.

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NOTE:- In case of Translation, we perform addition b/w matrices

NOTE:- For rotation & scaling, we perform multiplication b/w matrices.

eg - for 2 rotations $R_{45°}$ $R_{30°}$

$\downarrow$

$[O_1] \cdot [R_{45°}]$

$\Downarrow$

$[O_2]\ [R_{30°}]$

$= [O_3]$

This is tedious. To save computation time, so we can combine rotations.

matrix multiplication is not commutative

$\downarrow$

$\underbrace{[R_{45°}] \times [R_{30°}]}$

easy.

first find their product

$\downarrow$

R_1

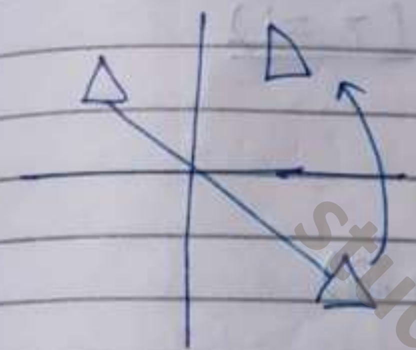
$[O_1] \times [R_1] = [O_2]$

- To perform all 3 rotation, scaling & translation, how can be form a combined Transformation matrix.

↓

we "homogenize", coordinates

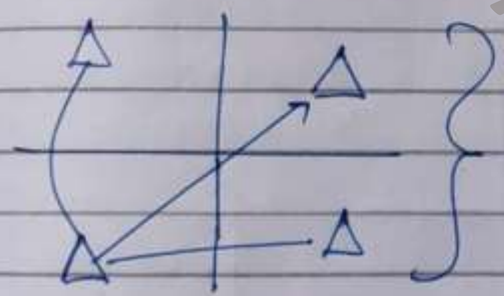
I



Translation (1)

(2) Anticlockwise rotation.

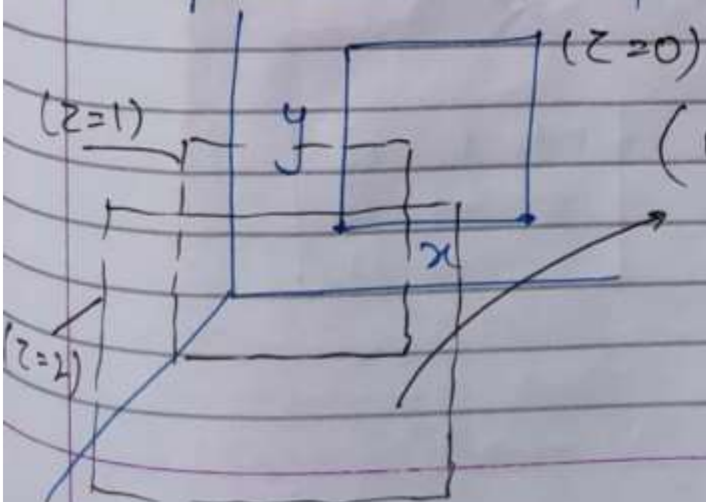
II



position can differ (first rotation then Translation).

- Homogeneous coordinates (to form composite Transformation matrix)

To homogenize 2-D coordinates, move 2-D plane in 3-D space.



(only 1 coordinate is fixed
pt. shifts in space with a diff. value)

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The pts. when combined lie in straight line as they are multiplied by scale of z.

$(X, Y) \longrightarrow (x, y, 1)$

3rd dimension
($z=1$)

At $z=2$

$(2x, 2y, 2)$

At $z=3 \longrightarrow (3x, 3y, 3)$