11 march, 19

Reflection

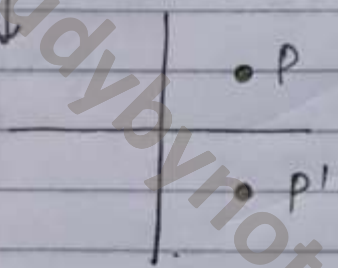
→ Order of operations matter, resultant position of object changes.

Reflection can be achieved by rotation also (180°)

↓

Axis of Reflection.

- x axis ↓



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

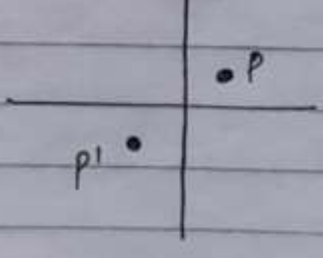
y coordinate becomes -ve.

- y axis. ↓

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

x coordinate becomes -ve.

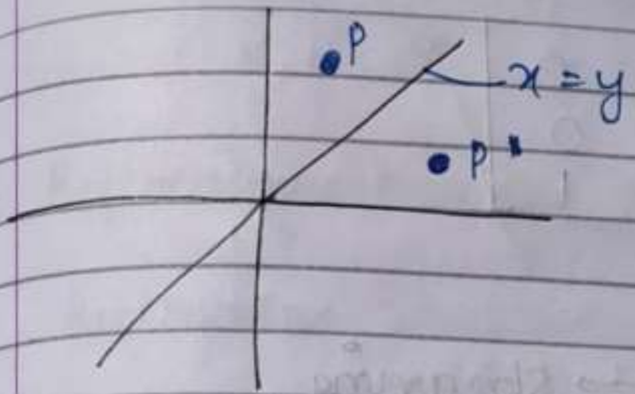
* Reflection abt. origin



$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

x & y → -ve

Reflection about a line



$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

x changes to y & y changes to x

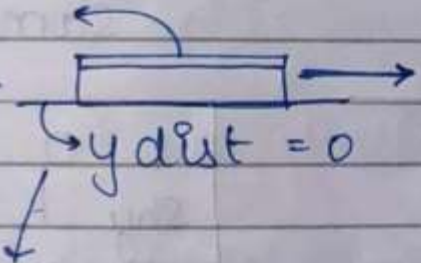
magnitude remains same.

* Shear **Transformation**

achieved using other Transformations.

- apply a force in x or y direction.

ydist = max. (most movement)



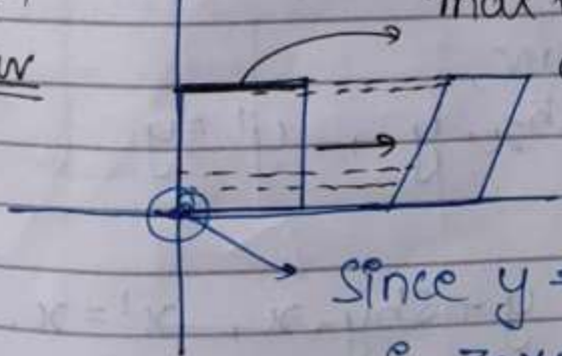
ydist = 0

Force applied ∝ dist of y.

Force ∝ x dist.

max movement (as max x)

x dir^n shear



max movement at this layer

since y = 0

∴ zero movement but all pts. will have equal movement

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- x dir^n] (y remains same)

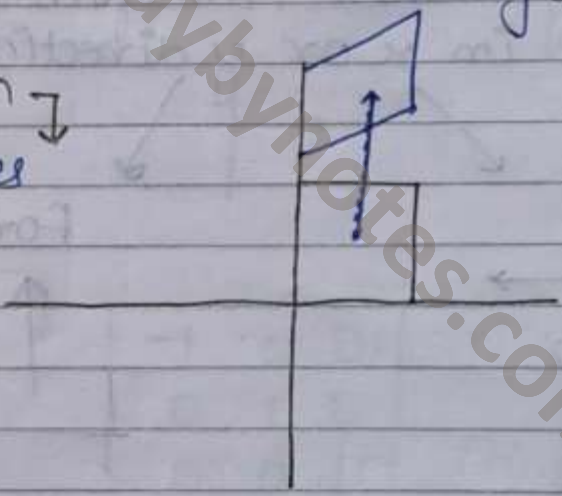
$$\begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x \rightarrow x + y\, sh_x$ → shearing factor (force applied in x dir^n)

used to apply force & pts. that are sheared depends on y dir^n.

- y dir^n]

(y coordinates change depending upon x)



(x coordinate remains same)

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$y \rightarrow y + x\, sh_y$.

NOTE:- For x direction shear,

$x' = x + sh_x \cdot y$, $y' = y$

NOTE:- For y dir^n shear,

$y' = y + sh_y \cdot x$, $x' = x$.

For both dir^n shear →

$$\begin{bmatrix} 1 & Sh_x & 0 \\ Sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{l} \rightarrow x \text{ factor} \\ \rightarrow y \text{ factor.} \\ \\ \end{array}$$

- Reflection Through an Arbitrary line :-

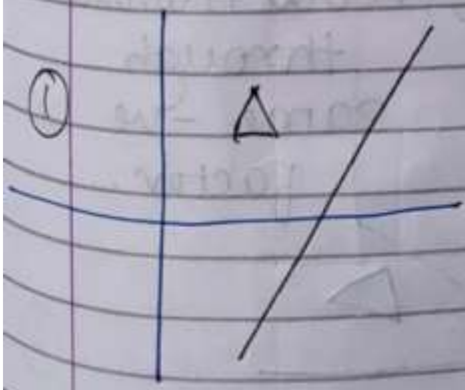
Reflection about y axis & x axis

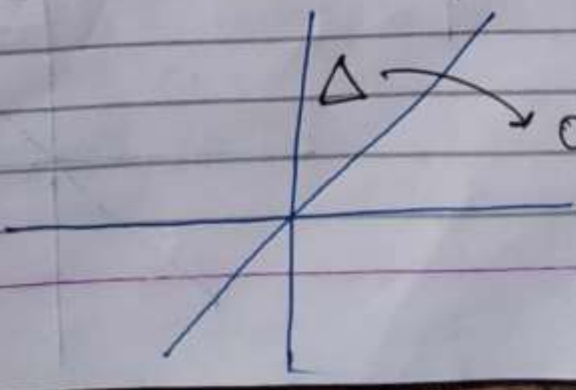
$$T_y = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad T_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Bring line to an axis for which we have direct reflection matrixes.

To coincide line with axis, we apply Transformations ∴ object will also change.

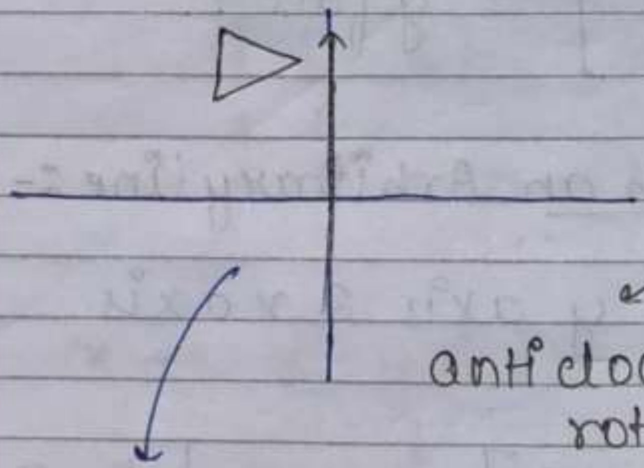
After that apply reflections & inverse reflections to make line at original pos.

①  ⟶ ② Translate line s.t it passes through origin.



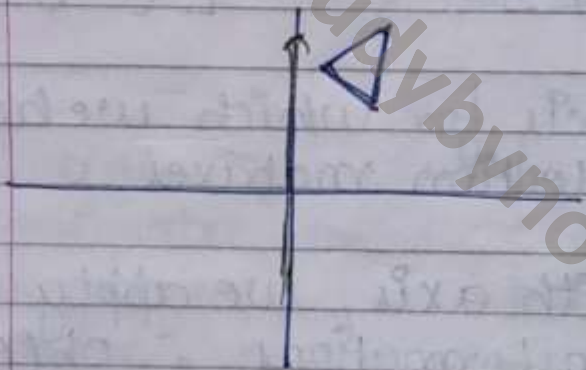
Obj. will also Translate

(3). Coincide line to one of the axis
rotate the line s.t
it coincides
to y axis.
(angle of rotation
depends on
eq^n of line).

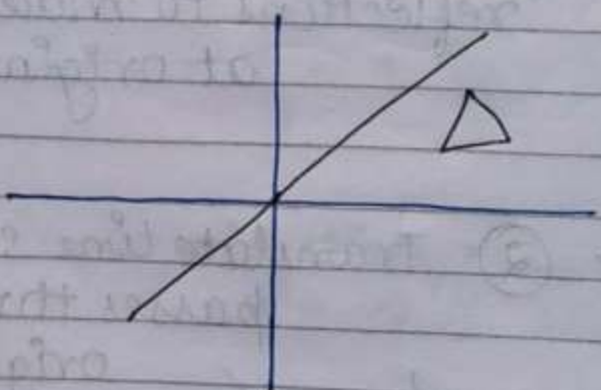


anticlockwise
rotation.

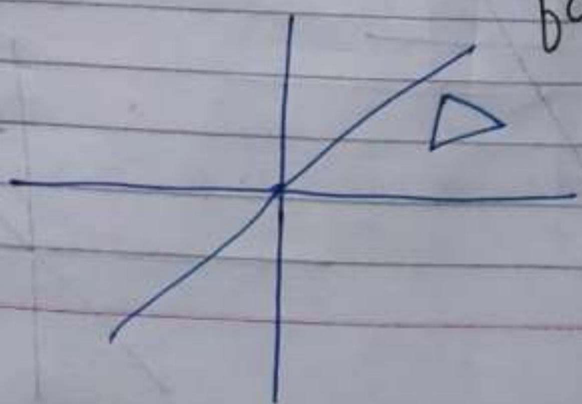
(4) Reflection abt y axis.



(5) Now reverse the reflections. (inverse
Transformation
↓
first rotate (~~aa~~ clockwise dir^n)



(6) Now translate
through
same -ve
factor.



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NOTE:- If a line is ∥ to one of the axis, just do Translation, no need of rotation.

For $y = 2$
Translate to $y = -2$, so as to bring it to origin.

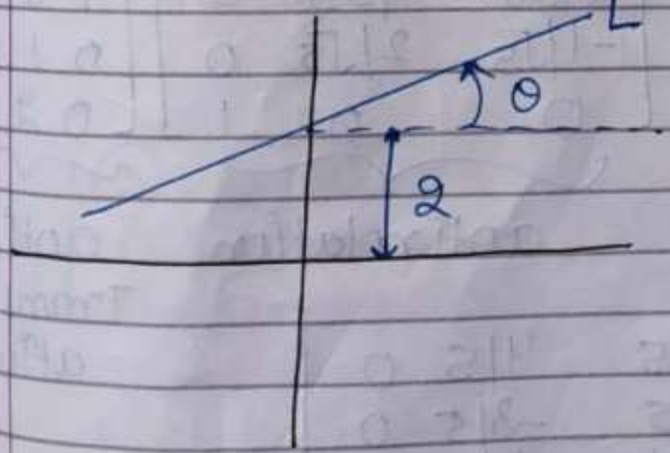
eg:- A Δ with vertices $(2,4,1)$, $(4,6,1)$ and $(2,6,1)$

line → $y = (x+4)/2$.
$y = \frac{x}{2} + 2$

$y = mx + c$
$\underline{m = 1/2}$

$\frac{1}{2} = m = \tan \theta$ → angle with x axis.

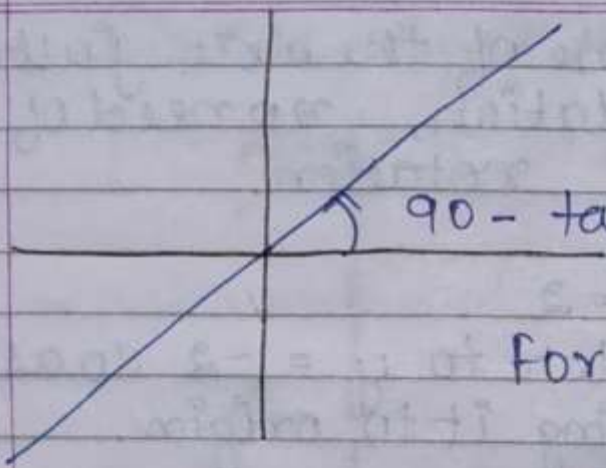
Intercept = 2



Slope $= \tan^{-1}(1/2)$
$= \underline{26.57^\circ}$

① Translate line to pass through origin
$(y = -2, x = 0)$

② L passes through origin.



$90 - \tan^{-1}(1/2) = \theta'$

For coinciding with x axis

$-(90 - 26.57^\circ)$

For coinciding with y axis]

$(90 - 26.57^\circ)$

$$\therefore T = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}}_{\text{Translation}} \underbrace{\begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} & 0 \\ 1/\sqrt{5} & +2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{rotation}} *$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{reflection}} \underbrace{\begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ -1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{antirotation}} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}}_{\text{anti Translation}}$$

$$= \begin{bmatrix} 3/5 & 4/5 & 0 \\ 4/5 & -3/5 & 0 \\ -8/5 & 16/5 & 1 \end{bmatrix}$$

$\therefore$ pre multiply obj.

$$\begin{bmatrix} 2 & 4 & 1 \\ 4 & 6 & 1 \\ 2 & 6 & 1 \end{bmatrix} T = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

* composite matrix

* Overall Scaling - (can be achieved through scaling matrix).

→ scaling an obj. uniformly in x & y dirⁿ.

→ If $S_x > S_y$ → no uniform scaling.

→ Changing coordinates uniformly in both dirⁿ.

→ $[x^* \; y^* \; h] = [x \; y \; 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & S \end{bmatrix}$

$= [x \; y \; S]$.

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

S_x, S_y

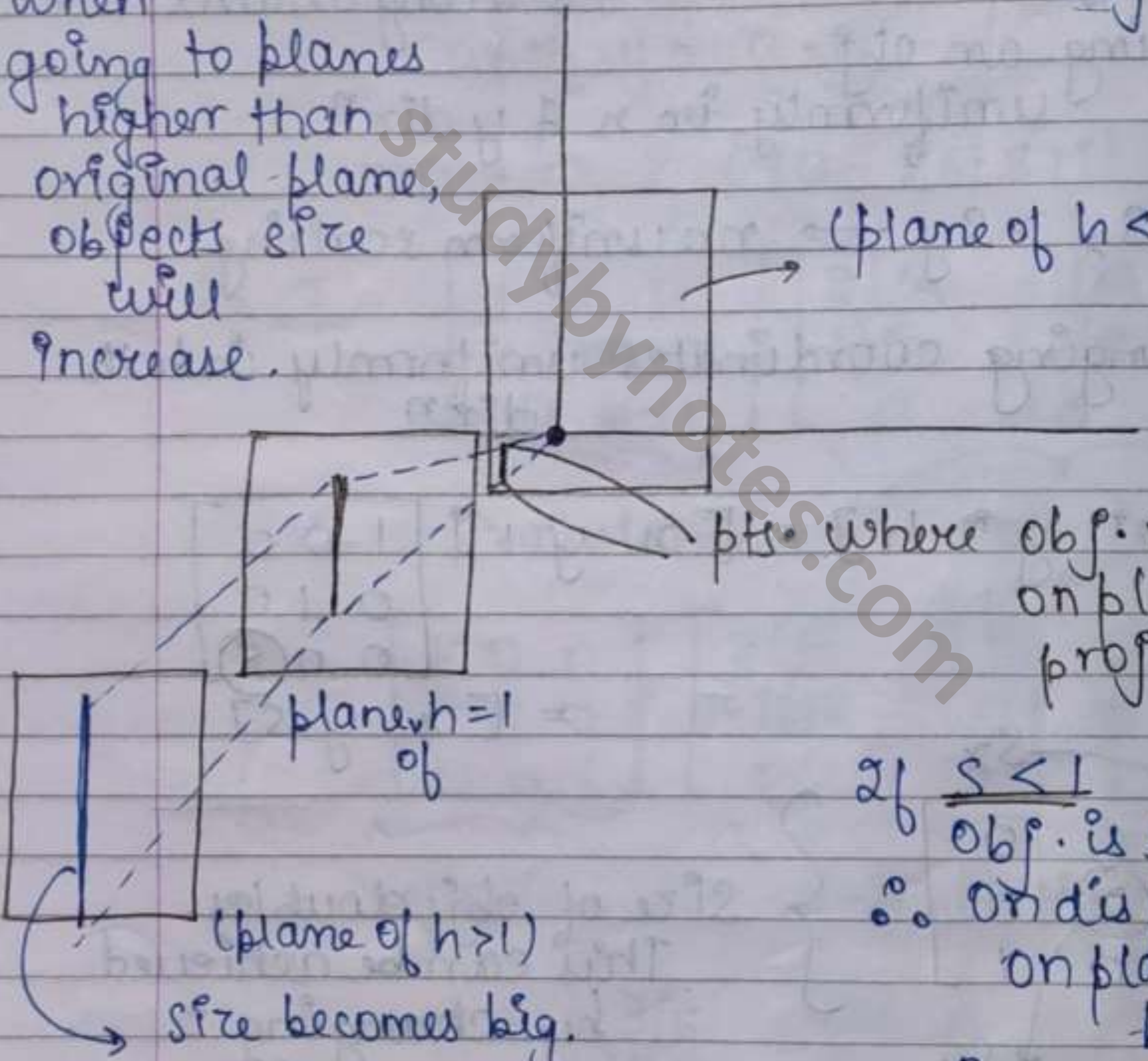
Size of obj. doubles. This can be achieved by changing z coordinate to a diff. value to make it 3-D.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & S \end{bmatrix}$$

changing this value is used for overall scaling.

Changing plane from 2-D to 3-D will affect size of object.

When going to planes higher than original plane, objects size will increase.



If $S < 1$

Obj. is small

∴ On displaying on plane h=1

size of obj. increases.

If $S > 1$

ultimate size of obj. reduces.

(as we need to represent on plane = 1)

∴ we are moving backwards.

NOTE:- S is a kind of reverse scaling factor

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & S \end{bmatrix} = \begin{bmatrix} 1/S & 0 & 0 \\ 0 & 1/S & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If $S = 2$ (>1)

$S_x = \frac{1}{2} = 0.5$ ($\therefore$ value reduces).

If $S = \frac{1}{2}$

$S_x = 2$ ($\therefore$ value increases).

* Classification of Transformations

rigid

- (shape of obj. doesn't change).
- distance preserving
- eg - Translation & rotation or uniform scaling.
- obj. is not distorted.
- intersecting lines remains intersecting.

Affine.

- parallelism preserving.
- Shape may change but || lines remains parallel
- eg - Translation, rotation, Scaling, Shearing.
- preserves parallelism & not lengths & angles.